Emma's Average Rate

Mathematical Goals

- Identify average rate of change using a piecewise linear graph.
- Describe average rate of change as speed that would be covered by piecewise graph connected end-to-end.
- Identify average rate of change when a piecewise linear function has positive y-intercept.
- Find an equation of a line through two points (in this case, the endpoints of a piecewise linear graph)

CCSSM Standards

- CCSS.MATH.CONTENT.HSF.IF.B.6
  Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Practice Standards

- SMP1: Make sense of problems and persevere in solving them
- SMP4: Model with mathematics

Mathematical Language Goals

- Describe processes about calculating average rate of change from distance-time graphs. Example: "I can find the average rate by using the two endpoints, dividing the difference in the y-coordinates by the difference in the x-coordinates."
- Make generalizations about multiplicative distance-time-average speed relationships in piecewise linear graphs. Example: "To find the average speed, I divide the total distance by the total time. The changes in the middle do not affect the result."

MLRs to support Language Goals

MLR2: Collect and Display

MLR5: Co-crafted Questions

MLR9: Co-Constructed Word Wall

MLR7: Compare and Connect

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Students Facing Goals
1. I can identify average rate of change from a piecewise linear graph. 2. I can interpret average rate of change in the context of a distance-time piecewise linear graph. 3. I can create an equation from a graph using two endpoints.

Lesson Glossary (Word Wall)	
Y-Intercept X-Intercept Slope Positive Negative Quadrant	Origin Steep Piecewise Linear Rate

Materials & Preparation

- Student Handout
  - Student Activity
  - Exit Slip
- Desmos Activity
  - For teachers *and* students

Math Notes For the Teacher

The main goal with this lesson is for students to develop understanding of average rate of change using a distance-time piecewise linear graph to motivate the calculation. The Desmos activity shows two characters, Emma who walks at a non-constant rate and "Average Emma" who always starts and ends in the same place and time as Emma, but who moves at a constant rate.

Note that even for piecewise graphs with non-constant speed, the average rate can always be found with $avg\ rate = \frac{total\ distance}{total\ time}$. This greatly simplifies finding average rate of a non-constant function (no need to do a weighted average!). In the immediate future, this formula corresponds with making the "slope triangle" in a linear graph and motivates writing a linear equation. (In case you are wondering, in the not-so distant future, students will also use this basic formula with the mean value theorem for derivatives!)

A secondary goal is for students to practice writing the equation of a line through two points, using the endpoints in the piecewise graph to get the slope (average rate of change) and a

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point. The students are encouraged to practice this when the left endpoint starts at (0,0), the easiest case that results in an equation y = mx, as well as when the left endpoint is not (0,0).

- The piecewise linear graph tracks the distance and time walked by Emma at different rates.
- Students are asked to consider another character, Average Emma, who starts at the same place and time as Emma, but walks at a constant pace.
- The average rate of change can be represented by Average Emma's constant pace.
- The average rate can be found by dividing the total distance by the total time.
- The main goal is for students to equate Emma's steady pace (constant rate) with the ratio of total distance to total time.

Introduction & Overview

1. Launch - Which One Doesn't Belong? (5 min)
2. Explore Part 1 - Emma's Average Rate
    a. What do you notice? What do you wonder? (3 min)
    b. Speed for Average Emma (8 min)
    c. Sharing students' solutions (5 min)
    d. Making connections about average rates (5 min)
3. Explore Part 2 - Generalizing Average Rate
    a. Work with the case of positive y-intercept (5 min)
    b. Sharing students' solutions and formalizing (5 min)
    c. Generalizing to the cases when the middle points/segments are moved (4 min)
4. Explore Part 2 - Emma's Equations
    a. Creating an equation (10 min)
    b. Sharing students' solutions and generalizing (5 min)
5. Summary - Exit Slip (5 min)

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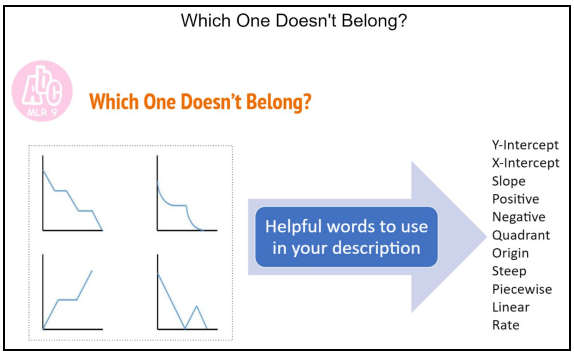
Phases of a Lesson

Notes for Teacher
Launch: Warm Up

Which One Doesn't Belong (WODB)? (5 min)

- The goal of this WODB is to cue students to start focusing thinking about piecewise graphs and their components. For example, students might say that the graph on the top right doesn't belong because it is the only graph with a curve. Or, the graph on the bottom right doesn't belong because it has two x-intercepts.
- "Co-constructed" Word Wall: For this activity students will be given a word bank with no definitions. There are two intentions with this: 1) Students might know these words from a prior class and be able to use them in the service of making a more precise explanation; We are not assuming this is where students will formally learn and define these terms; and 2) Students or as a class will construct new academic words as we go through the lesson.



Slide 2

Explore Part 1

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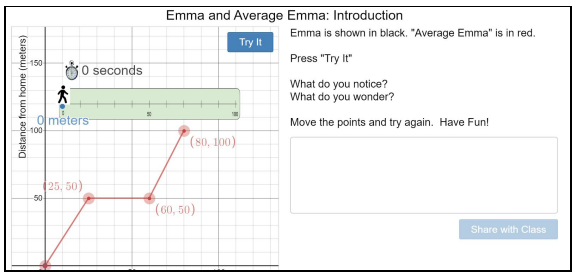
Emma's Average Rate

Note: WC=Whole Class and GW=Group Work

[GW/WC] Introduce the story about Emma and Average Emma. Give students time to press Try It and move points in slide 3, and ask them to answer question 1 ("What do you notice about the graph? What do you wonder?"). Be sure to give students time to play with the model for a few minutes. Note if they create an impossible graph, in this activity Desmos does not allow them to try it. After a few minutes, gather the class to collect and display observations or questions



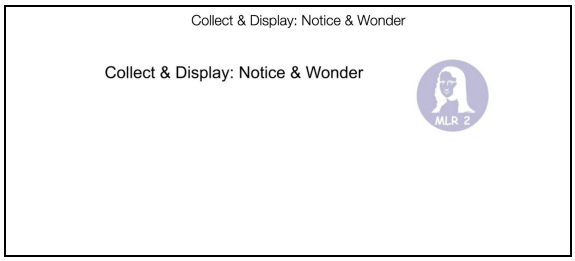
Slide 3

Some observations or questions might be:

- Emma and Average Emma both started and finished in at the same place and time.
- Emma and Average Emma walk at different rates. (unless you make Emma go the same pace the whole time!)
- How fast does Average Emma walk?
- Why does the graph disappear? (if the students create an impossible graph, the line segments disappear)

Note: It is critical that you take time to collect student's ideas and display them. This will give students a chance to reflect on the language practice of noticing and wondering about a situation.

Tie to MLR5: Students will likely craft questions from the task here.



Slide 4

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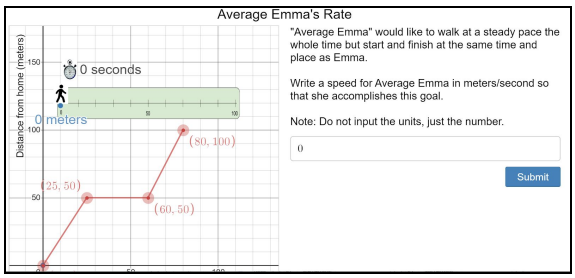
Allow students to solve the challenge on slide 5 (handout questions 2-3).

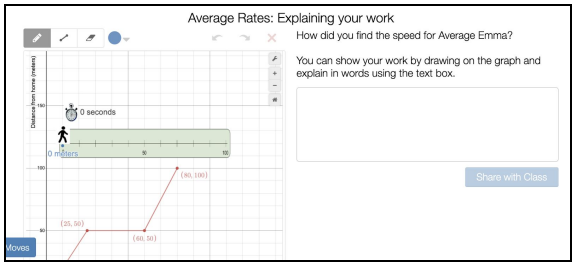
Notes

- The goal is for students to notice that Average Emma's rate is the ratio of Emma's **total distance** to **total time** (e.g., 100 meters per 80 seconds).
- This slide shows a somewhat easy case because the graph starts at (0,0).
- Watch out for students who take the average (arithmetic mean) of the slopes of each segment. This will result in an answer of 1.5. However, this is not Emma's average speed! This is a great point for discussion and clarifying the meaning of **average** in this context which is different from arithmetic mean



Slide 5

On Slide 6 students explain how they found the average rate for Emma. Note here it is useful to emphasize that although the calculator does not allow students to input units, the units on the speed are m/s.



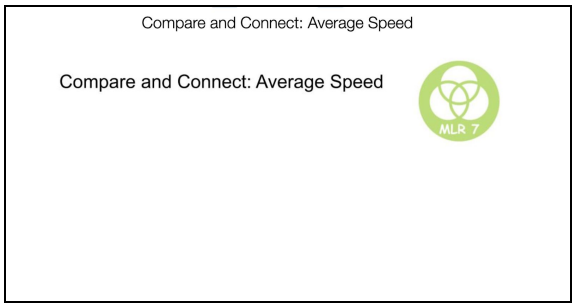
Slide 6

On Slide 7 you are prompted to Compare and Connect students' answers and explanations.

As part of this comparison and connecting, ask students to elaborate or respond to other students' solutions.

MLR 7

[WC/GW]] (5 min) Facilitate a whole-class discussion about the meaning of average rate of change.



Slide 7

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- Ask students: What is Emma's average speed? Why do we call it "average" speed? Note that this meaning of average is slightly different from the meaning students learned in statistics(!!)

Explore Part 2

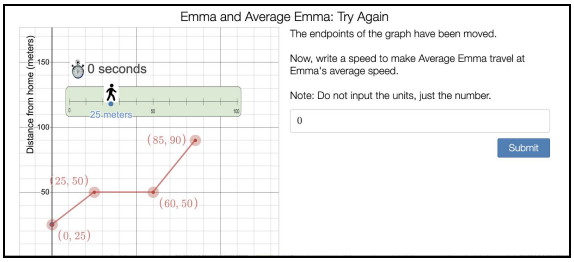
Generalizing Average Rate

[GW] (5 min) Let students work on slides 8 and 9 (questions 4-5). You might need to explain what "endpoints" mean if students get confused

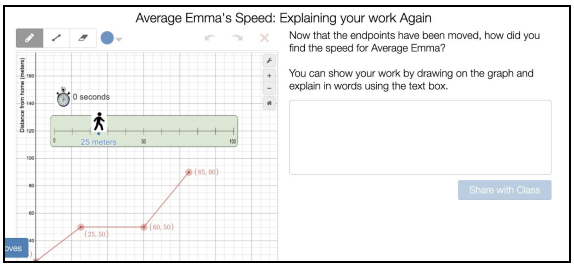
1) [WC] (5 min) Debrief students' solutions. Use students' solutions to generalize their work to a formula about average rate of change (e.g. difference in y-coordinates of endpoints divided by difference in x-coordinates because differences represent total time and total distance).

2) [WC] (5 min) Facilitate a whole-class discussion about what happens when you move the inner points/segments (but fix the endpoints). If nobody has made this generalization, lead students to realize that average rate of change is invariant in this case.

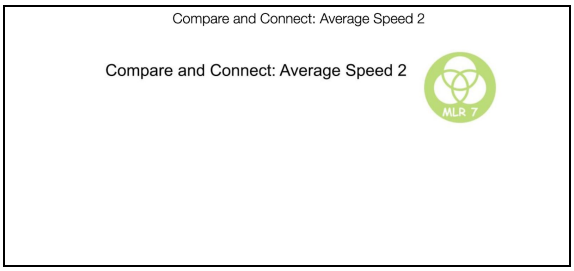
3) For fun, make the changeable Emma do crazy things :)
4) Slide 10 is there to remind you to **Compare and connect** students' ideas



Slide 8



Slide 9



Slide 10

Explore Part 3

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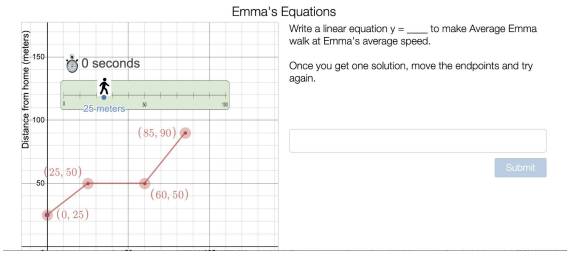
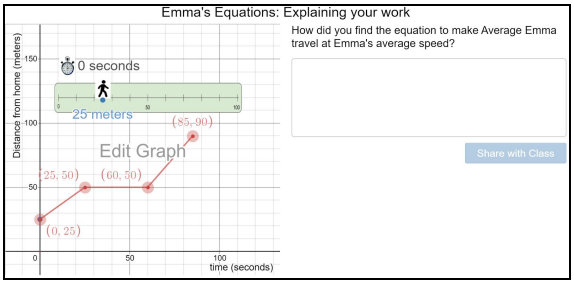
Emma's Equations (15 min)

5) [GW] (10 min) Let students work on slides 11-12 (questions 6-7). These questions ask students to create an equation of the form y= _____ to represent a walk at Emma's average rate in the more general case (when starting point is not the origin).



Slide 11



Slide 12

[WC] Debrief students' answers and explanations through Compare and Connect again.

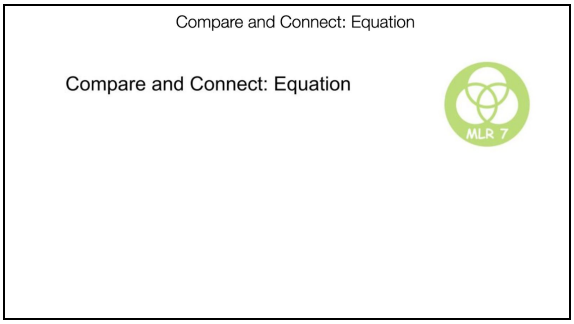
Building on students' solutions, generalize to the formula

y = Average Rate * x + starting point

or equivalently

y = (change in distance) / (change in time) *x + starting point.

There is a blank slide for notes if you want students to formally record their methods for solving the task



Slide 13

Slide 14

Summary

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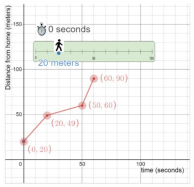
Exit Slip (5 min)

Collect the exit slip to gather assessment data. The Exit Slip consists of a piecewise linear distance-time graph, similar to those in the lesson - except that now there is only one character (Fernando). The students are asked to identify rate of change from the graph and to create a related equation and graph it.

Exit Slip

The graph below represents Fernando's walk from yesterday. Today Fernando walks the same path from yesterday and takes the same time but walks at a constant rate.

1) On the picture shown, sketch a graph to show Fernando's walk from today. Then write a sentence explaining how you made your graph correct.
2) What is Fernando's rate for today's walk? How do you know?
3) Write an equation of Fernando's walk from today. Explain your work.



Slide 15

Homework

No homework has been created yet.

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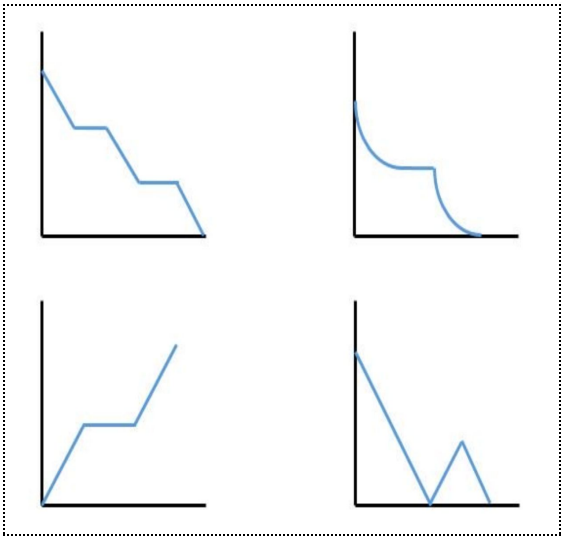
Name: _____ Group Number _____

Which One Doesn't Belong?

- ❏ Top Left
- ❏ Top Right
- ❏ Bottom Right
- ❏ Bottom Left



Explanation: The graph located at the _____ doesn't belong because_____

_____

_____

Here are some words you might want to use in your explanation:

Y-Intercept X-Intercept Slope Positive Negative Quadrant Quadrant	Origin Steep Piecewise Linear Rate

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Name: _____ Group Number _____

Emma's Average Rate

Desmos shows Emma in black and "Average Emma" in red. As you complete the desmos activities, answer the following questions.

1) Press the "Try It" button in desmos.

   (a) What do you notice?

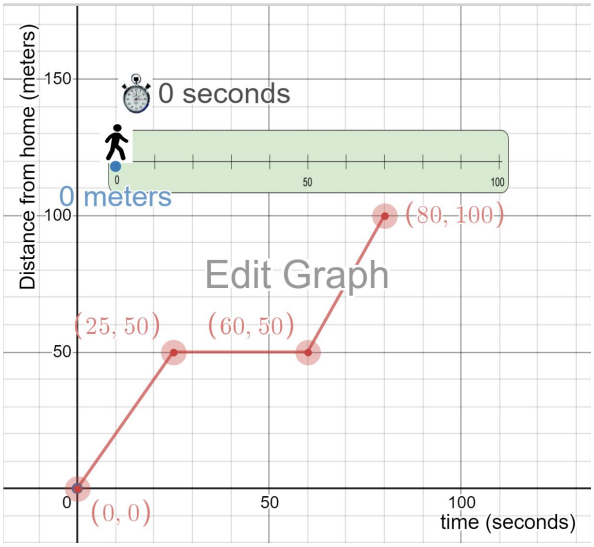
   _____

   _____

   (b) What do you wonder?

   _____

   _____



2) "Average Emma" would like to walk at a steady pace the whole time but start and finish at the same time and place as Emma. Write a speed for Average Emma in meters/second so that she accomplishes this goal.

   _____

3) How did you find the speed for Average Emma?

   _____

   _____

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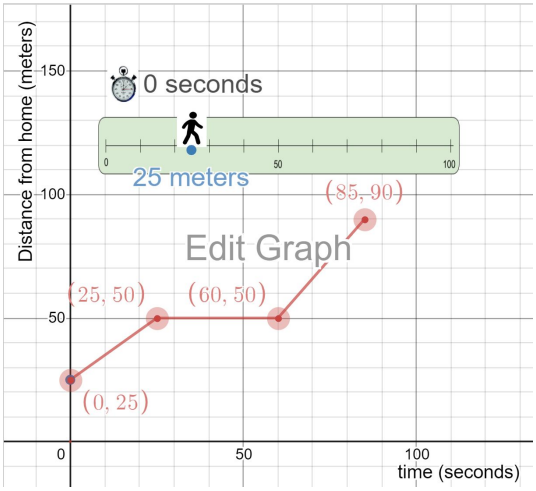
4) The endpoints of the graph have been moved. Now, write a speed to make Average Emma travel at Emma's average speed.

_____



5) Now that the endpoints have been moved, how did you find the speed for Average Emma?

_____

_____

6) Write a linear equation y = ____ to make Average Emma walk at Emma's average speed.

_____

7) How did you find the equation to make Average Emma travel at Emma's average speed?

_____

_____

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Name: _____ Group Number _____

Exit Slip

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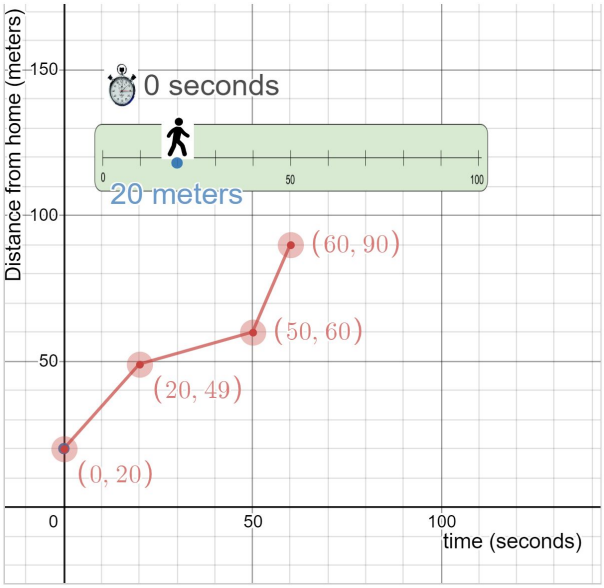

_____

_____

_____

_____

2) What is Fernando's rate for today's walk? How do you know?

_____

_____

3) Write an equation of Fernando's walk from today. Explain your work.

_____

_____

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