

Reflections

If you do not speak & read Spanish...

- How did you access the task?
- What resources did you use to engage?
- How did it feel to depend on colleagues?

If you do speak & read Spanish...

- How did you make sense of the task?
- What visual representations did you make?
- How did it feel to have colleagues depend on you?

A Critical Question

How can we teach rigorous mathematics to all students, including emergent bilingual students who are learning English as an additional language?



Language(s) & Learning Mathematics

Isn't 1+1 the same in any language?



Language(s) & Learning Mathematics

Common Core State Standards

Standards for Mathematical Practices

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Content

The image shows a document titled "Standards for Mathematical Content". It is organized into sections for different grade levels (e.g., Grade 1, Grade 2, Grade 3, Grade 4, Grade 5, Grade 6, Grade 7, Grade 8, Grade 9-12). Each grade level section contains a list of standards, often with sub-points. The document is presented in a vertical orientation.



Language(s) & Learning Mathematics

Common Core State Standards

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“Mathematically proficient students try to **communicate precisely to others**. They try to **use clear definitions in discussion** with others and in their own reasoning.... In the elementary grades, students **give carefully formulated explanations** to each other. By the time they reach high school they have learned to **examine claims and make explicit use of definitions.**”

Current Research

NSF CAREER Grant supporting 5 Year design and development study in 9th grade mathematics course in local high school

Our goal is to promote student participation and discussions. Today, we're going to highlight how we use language structures to promote equitable student engagement → Mathematical Language Routines (MLRs)













Mathematical Language Routines (MLRs)

Name	Description
 <p>Stronger and Clearer</p>	<p>What it is: It structured open instructional opportunity for students to make and defend their ideas and their work and other work.</p> <p>Example: Ask students to write an explanation that shows a claim. Then ask students to make their explanation after the discussion.</p>
 <p>Solved and Thriving</p>	<p>What it is: Capture students' and words and process in a table, collective reflection.</p> <p>Example: Use "notice and wonder" activity and record student process in a table.</p>
 <p>Critique, Revise, and Refine</p>	<p>What it is: Give students a piece of mathematical writing that is not their own to critique, reflect on, and revise.</p> <p>Example: Ask students to comment on what's correct and incorrect about a mathematical argument that a fellow student.</p>
 <p>Information Gap</p>	<p>What it is: Bring partners or team members different pieces of necessary information that must be used together to solve a problem.</p> <p>Example: Give students cards with different representations (e.g., graphs, tables, equations, stories). Ask each group to build the cards.</p>
 <p>Confronted Questions</p>	<p>What it is: Having students to generate mathematical questions for a situation. This creates space for students to surface the language of math questions.</p> <p>Example: Present a situation... "What do you notice and what do you wonder?" "What is your question?" Discuss. Then student is invited to solve all a class.</p>

 <p>Three Reads</p>	<p>What it is: A structured reading routine to ensure students access what they are being asked to do. Creates openings for reflection on what math questions are presented.</p> <p>Example: Read #1: What is the about? Read #2: What is the story? Read #3: What information do you need?</p>
 <p>Compare and Contrast</p>	<p>What it is: Compare and contrast different mathematical representations, examples, scenarios, and examples.</p> <p>Example: Ask students with different solutions to add, multiply. Then ask for them to reflect on similarities and differences.</p>
 <p>Discussion Supports</p>	<p>What it is: A set of job structures, routines, and tools for supporting participation and engagement.</p> <p>Example: Structured sharing in 6-partner sharing. Partner using discussion moves such as extend, repeat, agree, and like. Group and individual accountability.</p>
 <p>Co-Constructed Word Wall</p>	<p>What it is: A public display of words that all support a discussion. This set help students use academic language more fluently.</p> <p>Example: Students bring words that all support a discussion. Co-constructing graphics. Then use it for discussion. See Example "Thinking" activities.</p>
 <p>Noting at the Board</p>	<p>What it is: When a problem involves an action or a step, ask students to draw the step while introducing the problem or as the conclusion when the problem is solved.</p> <p>Example: Read a story problem about a race with a head start. Then ask students to run on a track and show the difference after between a runner that start with a head start and start. Then ask MLR.1 to compare the different being on track #2.</p>

Mathematical Language Routines (MLRs)

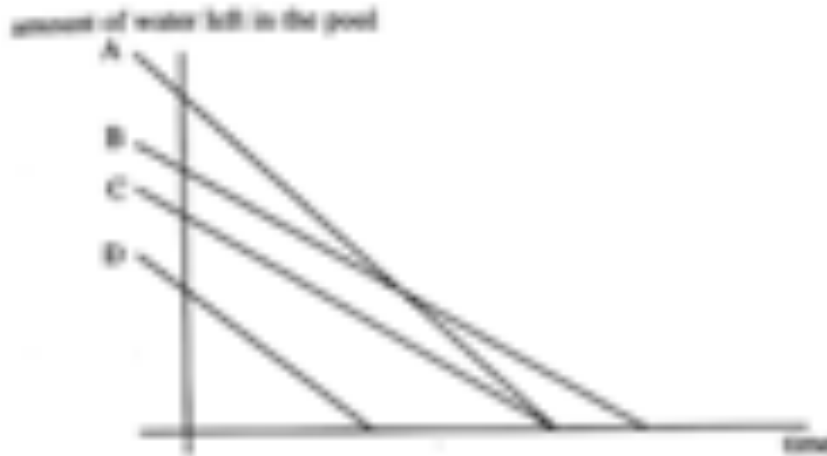
Name	Description
 Strong and Clear	<p>What it is: It structured and intentional opportunity for students to make and defend their ideas and their work with other people.</p> <p>Example: Ask students to write an explanation that shows as a class. Then ask students to make their explanation clear to someone else.</p>
 Collect and Display	<p>What it is: Capture students' work and display it in public, effective locations.</p> <p>Example: Use "notice and wonder" activity and record student answers on a poster.</p>
 Organize, Present, and Defend	<p>What it is: Give students a chance to organize their work and then use to explain, defend, and describe.</p> <p>Example: Ask students to organize an what's correct and incorrect about a mathematical argument that a fellow student.</p>
 Information Gap	<p>What it is: Bring partners or team members different pieces of necessary information that must be used together to solve a problem.</p> <p>Example: Give students cards with different representations (e.g., graphs, tables, equations, stories). Ask each group to build the cards.</p>
 Constructed Questions	<p>What it is: Asking students to generate mathematical questions for a situation. This creates space for students to explore the language of math questions.</p> <p>Example: Present a situation... "What if you asked and what do you wonder?" Write a question. Discuss. Then consider a number to solve all a class.</p>

 Three Reads	<p>What it is: A structured reading routine to ensure students access what they are being asked to do. Creates openings for reflection on what math questions are presented.</p> <p>Example: Read #1: What is the about? Read #2: What is the asking? Read #3: What information do you need?</p>
 Explain and Connect	<p>What it is: Compare and contrast different mathematical representations, examples, strategies, and language.</p> <p>Example: Ask students with different solutions to explain and justify. Then ask for how to effective strategies and connections.</p>
 Discussion Supports	<p>What it is: A set of job structures, routines, and tools for supporting participation and engagement.</p> <p>Example: Structured sharing in 6-minute sharing. Number using discussion moves such as discuss, speak your own, and like. Group accountability accountability.</p>
 Co-Constructed Word Wall	<p>What it is: A public space of words that are useful in discussing. This set help students use students' language their words.</p> <p>Example: Students bring words that is useful that it is discussing graphs. Then ask for the discussion. See Example "Thinking" activities.</p>
 Noting What	<p>What it is: When a problem involves an action or a story ask students to draw the story while considering the problem or to be considered when the problem is solved.</p> <p>Example: Read a story problem about a race with a runner. Then ask students to draw or to look and show the difference after between a runner that can't get a time that's that fast. Then ask MLR27 to connect the different being on another day.</p>

Example

Figure 8.1. An Algebra One problem.

Here are the graphs for the rates for four different pumps, which are emptying four different pools.



Explain all of your answers!

1. Which pool had the most water to start with?
2. Which pump completed its task first?
3. Which pump pumps the most water in a given time?

Download Rates

What do you notice? What do you wonder?



Download Rates

What do you notice? What do you wonder?



What do you notice?

-

What do you wonder?

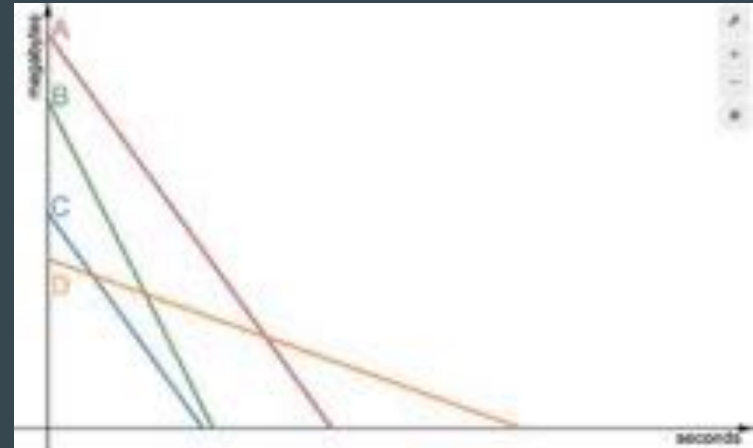
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Download Rates

Four friends disagree about whose phone is faster at downloading videos.

They test their phones by downloading different videos. Each friend writes down how large the video file is and how much time the phone used to download the video.

They use their information to make a graph showing the time each phone used to download a video. The phones are labeled A, B, C, and D.



Sample Tasks



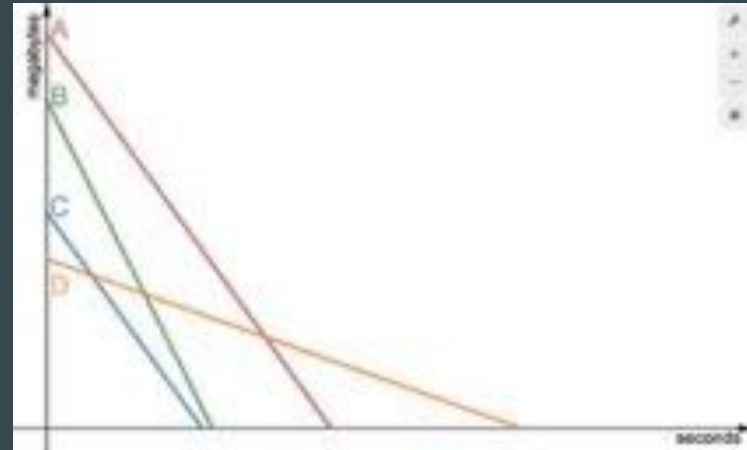
P. ###

1) Which phone downloaded the biggest video file? Where do you see this on the graph?

2) Which phone completed the download in the smallest amount of time? Where do you see this on the graph?

3) Which phone downloads fastest? How do you know?

4) What else do you notice about this graph?



Gallery Walk



P. ###

- Please view some of the sample responses on Jamboard link (see handout, p. 8)
- Then edit your solution, as needed.

Four lines represent the download rates of four different computers.

The graph shows the amount of data downloaded (in megabytes) over time (in seconds).

The x-axis is labeled "time" and the y-axis is labeled "data downloaded".

Line C is obviously the slowest. It downloaded the least number of megabytes but took the longest in seconds.

Lines A and C look like they're the same, so I think they both downloaded at the same rate but C just had less megabytes in the beginning. But I think it's line B because it downloaded almost as much as A, but finished around the time at B.

Four lines represent the download rates of four different computers.

The graph shows the amount of data downloaded (in megabytes) over time (in seconds).

The x-axis is labeled "time" and the y-axis is labeled "data downloaded".

$13.3/9.5 = 1.4 \text{ mb/s}$

$11/5.5 = 2 \text{ mb/s}$ Fastest

$7/5 = 1.4 \text{ mb/s}$

$5.7/10 = 0.57 \text{ mb/s}$ Slowest

Line B was the fastest because it has the highest rate of change.

Whole Class Discussion

- Where did you see language structures in this presentation?

Whole Class Discussion

- Where did you see language structures in this presentation?
 - Notice and wonder (Co-Crafted Questions)
 - Modified gallery walk (Collect and Display)
 - Alternating between small group and whole class (Discussion Support)
- How can you bring this back to your school site?

<https://meld.sdsu.edu/>

Selected References

- Celedon-Pattichis, S., & Ramirez, N. G. (2012). *Beyond good teaching : advancing mathematics education for ELLs*. Reston, VA: National Council of Teachers of Mathematics.
- Chval, K. B., & Chávez, Ó. (2012). Designing Math Lessons for English Language Learners. *Mathematics Teaching in the Middle School*, 17(5), 261–265. <https://doi.org/10.5951/mathteacmidscho.17.5.0261>
- Coggins, D., Kravin, D., Coates, G. D., & Carroll, M. D. (2014). *English language learners in the mathematics classroom*. Retrieved from <https://www.overdrive.com/search?q=A301EEAD-14A4-49A4-9EF8-7F2E58AE21C4>
- Driscoll, M., Nikula, J., & DePiper, J. (2016). *Mathematical thinking and communication: access for English learners*. Portsmouth, NH: Heinemann.
- Moschkovich, J. N. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics*, 19, 11–19.
- Moschkovich, J., & Zahner, W. (2018). Using the academic literacy in mathematics framework to uncover multiple aspects of activity during peer mathematical discussions. *ZDM*. <https://doi.org/10.1007/s11858-018-0982-9>
- Moschkovich, J. (2015). Academic literacy in mathematics for English Learners. *The Journal of Mathematical Behavior*, 40, 43–62. <https://doi.org/10.1016/j.jmathb.2015.01.005>
- Walqui, A. (2006). Scaffolding Instruction for English Language Learners: A Conceptual Framework. *International Journal of Bilingual Education and Bilingualism*, 9(2), 159–180. <https://doi.org/10.1080/13670050608668639>
- Zahner, W. (2012). ELLs and group work: It can be done well. *Mathematics Teaching in the Middle School*, 18, 156–164.
- Zahner, W., Velazquez, G., Moschkovich, J. N., Vahey, P., & Lara-Meloy, T. (2012). Mathematics teaching practices with technology that support conceptual understanding for Latino/a students. *Journal of Mathematical Behavior*, 31, 431–446. <https://doi.org/10.1016/j.jmathb.2012.06.002>
- Zahner, William, Milbourne, H., & Wynn, L. (2018). Developing and refining a framework for mathematical and linguistic complexity in tasks related to rates of change. *The Journal of Mathematical Behavior*. <https://doi.org/10.1016/j.jmathb.2018.04.003>

